Mathematics of Duplicate Bridge Tournaments

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Part I – The Mitchell movement and its derivatives

Part II – Scoring at duplicate bridge

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Introduction

The commonest form of bridge competition is the pairs contest, where the unit is the partnership, and each partnership competes against other partnerships. In order that the contest should be fair, and not dependent on the random element of the deal, the same hands are played several times – after playing to a trick, players do not mix their cards together, but place them face down in front of them. At the end of the hand, the cards are replaced in slots (labelled North, South, East, West) in a ‘board’, to be replayed elsewhere. At the end of a round, which usually consists of two, three or four boards, boards are moved to another table; players likewise move so that they encounter fresh opponents on the next round.

A typical session in a bridge club lasts 3 to 3½ hours, during which there is time to play 24 to 28 boards. The competition is controlled by a tournament director, who is frequently himself a competitor, and who has to ensure that players and boards move correctly after each round. His task is eased and mistakes are less likely to occur if the movements are devised to be as simple as possible – cyclic and preferably one table at a time.

Part I : The Mitchell movement and its derivatives

The simplest form of movement is the Mitchell movement, as exemplified in Table I for the case of 7 tables, 14 pairs.

The tables are numbered 1 to 7, and the pairs are number 1 to 14. The boards are designated by the letters *A* to *G*, each letter representing a set of *k* boards, the value of *k* being usually 2, 3 or 4. Pairs 1 to 7 remain stationary at their own tables and play NS (North-South). Pairs 8 to 14 play EW, and move cyclically up one table each round. The boards are moved cyclically down one table each round.

Table I indicates the NS pair number, the board letter and the EW pair number on each table each round. (The NS pair numbers are repeated for the sake of uniformity – in other types of movement the NS pairs may change their positions.) For example, at table 2 in the first round, boards *B* are played by pair 2 sitting NS against pair 9 sitting EW.

If there are only 13 pairs competing, the final column of Table I is omitted; there is no pair 7, and the moving pairs have to sit out for one round.

Table I
7-Table Mitchell

Round	Table Number						
	1	2	3	4	5	6	7
1	1 *A* 8	2 *B* 9	3 *C* 10	4 *D* 11	5 *E* 12	6 *F* 13	7 *G* 14
2	1 *B* 14	2 *C* 8	3 *D* 9	4 *E* 10	5 *F* 11	6 *G* 12	7 *A* 13
3	1 *C* 13	2 *D* 14	3 *E* 8	4 *F* 9	5 *G* 10	6 *A* 11	7 *B* 12
4	1 *D* 12	2 *E* 13	3 *F* 14	4 *G* 8	5 *A* 9	6 *B* 10	7 *C* 11
5	1 *E* 11	2 *F* 12	3 *G* 13	4 *A* 14	5 *B* 8	6 *C* 9	7 *D* 10
6	1 *F* 10	2 *G* 11	3 *A* 12	4 *B* 13	5 *C* 14	6 *D* 8	7 *E* 9
7	1 *G* 9	2 *A* 10	3 *B* 11	4 *C* 12	5 *D* 13	6 *E* 14	7 *F* 8
Key: 1 *A* 8, where 1 is the NS pair number, *A* is the board and 8 is the EW pair number.							

With an even number of tables (e.g. 8) the Mitchell movement is different in structure (Table IIa). Here tables 1 and 8 *share* boards, possible because the symbols *A, B* etc each represent a set of *k* boards, and *k* is normally greater than 1. Between tables 4 and 5 there is a set of boards (A *relay*) which is out of play for one round.

Table IIa
8-Table Relay and Share Mitchell

	Table Number																							
Round	1			2			3			4			5			6			7			8		
1	1	*A*	9	2	*B*	10	3	*C*	11	4	*D*	12	5	*F*	13	6	*G*	14	7	*H*	15	8	*A*	16
2	1	*B*	16	2	*C*	9	3	*D*	10	4	*E*	11	5	*G*	12	6	*H*	13	7	*A*	14	8	*B*	15
3	1	*C*	15	2	*D*	16	3	*E*	9	4	*F*	10	5	*H*	11	6	*A*	12	7	*B*	13	8	*C*	14
4	1	*D*	14	2	*E*	15	3	*F*	16	4	*G*	9	5	*A*	10	6	*B*	11	7	*C*	12	8	*D*	13
5	1	*E*	13	2	*F*	14	3	*G*	15	4	*H*	16	5	*B*	9	6	*C*	10	7	*D*	11	8	*E*	12
6	1	*F*	12	2	*G*	13	3	*H*	14	4	*A*	15	5	*C*	16	6	*D*	9	7	*E*	10	8	*F*	11
7	1	*G*	11	2	*H*	12	3	*A*	13	4	*B*	14	5	*D*	15	6	*E*	16	7	*F*	9	8	*G*	10
8	1	*H*	10	2	*A*	11	3	*B*	12	4	*C*	13	5	*E*	14	6	*F*	15	7	*G*	16	8	*H*	9

Rules for the construction of tables

The numbers representing moving pairs and the letters representing boards form a Graeco-Latin square in Table I, and indeed the rules governing the construction of Mitchell-type movements are similar to, but not identical with, those governing the construction of Graeco-Latin squares. The rules are as follows.

(1A) No letter may appear more than once in the same column. (A stationary pair may not play the same boards more than once.)

(1B) No number may appear more than once in the same row. (A moving pair cannot play at more than one table at a time.)

(2) No combination of letter and number may appear more than once. (A moving pair may not play the same boards more than once.)

A *complete* movement is defined as one in which all the pairs play all the boards. A complete movement must also obey rules (3A) and (3B).

(3A) Each column must contain a complete permutation of the letters.

(3B) Each row must contain a complete permutation of the numbers.

Although complete movements are preferred to incomplete ones, there are many occasions when incomplete movements must be played, e.g. when there is an odd number of pairs or when time runs short and the movement has to be curtailed.

There are two further rules which are in the nature of mild recommendations, but may, however, be broken.

(4A) A number should not appear more than once in the same column. (Pairs should not encounter the same opponents more than once.)

(4B) A letter should not appear more than once in the same row. (Boards should not have to be shared between tables.)

Table I obeys all seven rules; Table IIa obeys all except (4B).

Standard form

These rules permit any permutations of rows and columns, and any relabelling of EW pair numbers and boards to be made. In order to allow the movement to be easily controlled, tables are wherever possible expressed in standard form thus:

- The first row contains the sets of boards *A, B, C,* etc in order, with allowance for sharing tables and relays.
- The first row contain the EW pair numbers in ascending order.
- In each subsequent row, the board symbols are displaced one column to the left, and the EW pair numbers are displaced one column to the right (cyclically).

The advantages of the standard form are as follows:

- The first round is easy to set out.
- Boards move cyclically down one table after each round.
- Moving pairs move cyclically up one table after each round.
- Stationary pairs play the boards in sequence.

Duality

We can establish a duality by exchanging rows with columns, and letters with numbers of moving pairs. Evidently Rules (1A) and (1B), (3A) and (3B), (4A) and (4B) are dual pairs, while Rule (2) is self-dual.

Hence corresponding to any Mitchell-type movement, there exists a dual movement obtained by interchanging rows with columns and letters with numbers. Moreover the dual of a complete movement is a complete movement.

Table IIb
Skip movement for 8 tables

Round	Table Number																							
	1			2			3			4			5			6			7			8		
1	1	*A*	9	2	*B*	10	3	*C*	11	4	*D*	12	5	*E*	13	6	*F*	14	7	*G*	15	8	*H*	16
2	1	*B*	16	2	*C*	9	3	*D*	10	4	*E*	11	5	*F*	12	6	*G*	13	7	*H*	14	8	*A*	15
3	1	*C*	15	2	*D*	16	3	*E*	9	4	*F*	10	5	*G*	11	6	*H*	12	7	*A*	13	8	*B*	14
4	1	*D*	14	2	*E*	15	3	*F*	16	4	*G*	9	5	*H*	10	6	*A*	11	7	*B*	12	8	*C*	13
5	1	*E*	12	2	*F*	13	3	*G*	14	4	*H*	15	5	*A*	16	6	*B*	9	7	*C*	10	8	*D*	11
6	1	*F*	11	2	*G*	12	3	*H*	13	4	*A*	14	5	*B*	15	6	*C*	16	7	*D*	9	8	*E*	10
7	1	*G*	10	2	*H*	11	3	*A*	12	4	*B*	13	5	*C*	14	6	*D*	15	7	*E*	16	8	*F*	9
8	1	*H*	9	2	*A*	10	3	*B*	11	4	*C*	12	5	*D*	13	6	*E*	14	7	*F*	15	8	*G*	16

Table I is self-dual. Table IIb is the dual of Table IIa.
Note: Table IIa may be converted into a valid skip movement by the following transformations:

1. Interchange 9, 10, 11, 12, 13, 14, 15, 16 with *A, B, C, D, E, F, G, H,* respectively.
2. Transpose rows and columns.

In addition to convert to Table IIb, which is in standard form, two additional transformations must be made:

3. Reverse the order of the rows.
4. Reletter the boards *A, B, C, D, E, F, G, H* as *H, G, F, E, D, C, B, A, respectively.*

The duality between Tables IIa and IIb shows up in the following features:

- Whereas in Table IIa there is a relay between tables 4 and 5, so in Table IIb the EW pairs make a double move after round 4.
- Whereas in Table IIa, boards are shared between tables 1 and 8, so in Table IIb pairs have the same opponents in rounds 1 and 8.

Rectangular movements

Tables need not be square. Table IIIa shows a 'rover' movement for 7 rounds and 8 tables, and is typical of movements for $T - 1$ rounds and T tables, where T is of the form $6m$ or $6m + 2$.

Table IIIa
Rover Movement

Round	Table Number																							
	1			2			3			4			5			6			7			8		
1	1	*A*	16	2	*B*	10	3	*C*	11	4	*D*	12	5	*E*	13	6	*F*	14	7	*G*	15	8	*A*	9
2	1	*B*	15	2	*C*	9	3	*D*	16	4	*E*	11	5	*F*	12	6	*G*	13	7	*A*	14	8	*D*	10
3	1	*C*	14	2	*D*	15	3	*E*	9	4	*F*	10	5	*G*	16	6	*A*	12	7	*B*	13	8	*G*	11
4	1	*D*	13	2	*E*	14	3	*F*	15	4	*G*	9	5	*A*	10	6	*B*	11	7	*C*	16	8	*C*	12
5	1	*E*	12	2	*F*	16	3	*G*	14	4	*A*	15	5	*B*	9	6	*C*	10	7	*D*	11	8	*F*	13
6	1	*F*	11	2	*G*	12	3	*A*	13	4	*B*	16	5	*C*	15	6	*D*	9	7	*E*	10	8	*B*	14
7	1	*G*	10	2	*A*	11	3	*B*	12	4	*C*	13	5	*D*	14	6	*E*	16	7	*F*	9	8	*E*	15

Pair 16 is the rover pair and moves up two tables round the circuit 1 to 7. All other EW pairs move up one table at a time, except when they are displaced by the rover, when they go to table 8 for one round before resuming their normal progress. Table 8 shares boards with the table occupied by the rover.

The dual of the rover is the *Worger* movement, displayed in Table IIIb. Here there is a roving set of boards (*X*), which move two tables at a time, displacing the ‘ordinary’ boards. On the final round, moving pairs return to the table where they played *X*, and play the ordinary boards which they missed.

Table IIIb
Worger Movement

	Table Number																				
Round	1			2			3			4			5			6			7		
1	1	*A*	8	2	*B*	9	3	*C*	10	4	*D*	11	5	*E*	12	6	*F*	13	7	*X*	14
2	1	*B*	14	2	*C*	8	3	*D*	9	4	*E*	10	5	*X*	11	6	*G*	12	7	*A*	13
3	1	*C*	13	2	*D*	14	3	*X*	8	4	*F*	9	5	*G*	10	6	*A*	11	7	*B*	12
4	1	*X*	12	2	*E*	13	3	*F*	14	4	*G*	8	5	*A*	9	6	*B*	10	7	*C*	11
5	1	*E*	11	2	*F*	12	3	*G*	13	4	*A*	14	5	*B*	8	6	*X*	9	7	*D*	10
6	1	*F*	10	2	*G*	11	3	*A*	12	4	*X*	13	5	*C*	14	6	*D*	8	7	*E*	9
7	1	*G*	9	2	*X*	10	3	*B*	11	4	*C*	12	5	*D*	13	6	*E*	14	7	*F*	8
8	1	*D*	12	2	*A*	10	3	*E*	8	4	*B*	13	5	*F*	11	6	*C*	9	7	*G*	14

Note that the rover movement violates Rule (4B), whereas the Worger Movement violates Rule (4A).

Extended Mitchell

It may be convenient for the number of rounds to be two greater than the number of tables *T*; for example, when *T* = 10 and 24 boards are to be played. On such occasions an extended Mitchell may be used. There are two cases, depending on whether *T* is odd or even. Tables IV and V show the movements when *T* = 7 and *T* = 8 respectively.

Table IV
Extended Mitchell – Odd number of tables

	Table Number																				
Round	1			2			3			4			5			6			7		
1	1	*A*	8	2	*B*	9	3	*C*	10	4	*D*	11	5	*E*	12	6	*F*	13	7	*G*	14
2	1	*B*	14	2	*C*	8	3	*D*	9	4	*E*	10	5	*F*	11	6	*G*	12	7	*H*	13
3	1	*C*	13	2	*D*	14	3	*E*	8	4	*F*	9	5	*G*	10	6	*H*	11	7	*I*	12
4	1	*X*	12	2	*E*	13	3	*F*	14	4	*G*	8	5	*H*	9	6	*I*	10	7	*A*	11
5	1	*E*	11	2	*F*	12	3	*G*	13	4	*H*	14	5	*I*	8	6	*A*	9	7	*B*	10
6	1	*F*	10	2	*G*	11	3	*H*	12	4	*I*	13	5	*A*	14	6	*B*	8	7	*C*	9
7	1	*G*	9	2	*H*	10	3	*I*	11	4	*A*	12	5	*B*	13	6	*C*	14	7	*D*	8
8	1	*H*	8	2	*I*	9	3	*A*	10	4	*B*	11	5	*C*	12	6	*D*	13	7	*E*	14
9	1	*I*	14	2	*A*	13	3	*B*	12	4	*C*	11	5	*D*	10	6	*E*	9	7	*F*	8

Table IVa

Round																					
8	1	*H*	8	2	*I*	9	3	*A*	10	4	*C*	11	5	*C*	12	6	*D*	13	7	*E*	14
9	1	*I*	14	2	*A*	13	3	*B*	11	4	*B*	12	5	*D*	10	6	*E*	9	7	*F*	8

Note that in Table IV pair 4 encounters pair 11 on three occasions; this constitutes a particularly flagrant violation of Rule (4A). For this reason. The final two rounds are usually modified as shown in Table IVa.

Table V
Extended Mitchell – Even Number of Tables

Round	Table Number																							
	1			2			3			4			5			6			7			8		
1	1	*A*	9	2	*B*	10	3	*C*	11	4	*D*	12	5	*F*	13	6	*G*	14	7	*H*	15	8	*I*	16
2	1	*B*	16	2	*C*	9	3	*D*	10	4	*E*	11	5	*G*	12	6	*H*	13	7	*I*	14	8	*J*	15
3	1	*C*	15	2	*D*	16	3	*E*	9	4	*F*	10	5	*H*	11	6	*I*	12	7	*J*	13	8	*A*	14
4	1	*D*	14	2	*E*	15	3	*F*	16	4	*G*	9	5	*I*	10	6	*J*	11	7	*A*	12	8	*B*	13
5	1	*E*	13	2	*F*	14	3	*G*	15	4	*H*	16	5	*J*	9	6	*A*	10	7	*B*	11	8	*C*	12
6	1	*F*	12	2	*G*	13	3	*H*	14	4	*I*	15	5	*A*	16	6	*B*	9	7	*C*	10	8	*D*	11
7	1	*G*	11	2	*H*	12	3	*I*	13	4	*J*	14	5	*B*	15	6	*C*	16	7	*D*	9	8	*E*	10
8	1	*H*	10	2	*I*	11	3	*J*	12	4	*A*	13	5	*C*	14	6	*D*	15	7	*E*	16	8	*F*	9
9	1	*I*	9	2	*J*	10	3	*A*	11	4	*B*	12	5	*D*	13	6	*E*	14	7	*F*	15	8	*G*	16
10	1	*J*	16	2	*A*	15	3	*B*	14	4	*C*	13	5	*E*	12	6	*F*	11	7	*G*	10	8	*H*	9

Note that in Table V there is a relay between tables 4 and 5, as well as between tables 8 and 1.

Bowman movements

The duals of the extended Mitchells may be used when the number of rounds is two fewer than the number of tables. These are known as Bowman[*] movements. The dual of Table IV involves sharing boards amongst three tables on the fourth round; which can be avoided by means of a dual modification.

[*] Described in *EBU Manual of Duplicate Bridge Movements* – PP 17, 19 and 21.

Part II – Scoring at duplicate bridge

The result of each hand is scored as in rubber bridge, with the following exceptions.

(a) There is no distinction between 'above the line' and 'below the line'.
(b) Honours do not count.
(c) There is a bonus of 50 points for fulfilling a part-score contract, 300 for a non-vulnerable game and 500 for a vulnerable game.
(d) Part-scores are not carried over from one hand to another. Vulnerability is printed on the board.

The scores are subsequently converted to *match points*, which are equivalent to ranks. Each board carries with it a travelling score slip, on which the result is entered every time the board is played. At the end of the competition, each score slip is match pointed. For example, the score slip for the first board of set *A* played according to Table I might be as shown below.

						Match Points	
NS	EW	Contract	By	Tricks	Score to NS	NS	EW
1	8	2♦ - 1	N	7	-50	7	5
2	10	2♦ - 2	N	6	-100	2	10
3	12	2♣ ✓	E	8	-90	4	8
4	14	2♦ ✓	N	8	+90	12	0
5	9	3♣ ✓	E	9	-110	0	12
6	11	2♦ - 1	N	7	-50	7	5
7	13	2♠ - 1	W	7	+50	10	2

The scores are ranked: the NS pair with the lowest score (-110) is awarded no match points; the next lowest, two match points, and so on until the NS pair with the highest score (+90) receives $2(n – 1)$ match points (a 'top') where n is the number of times the board has been played. The factor of 2 is included to avoid fractions in the case of ties, for example the two scores of -50 as above. The EW pairs score the complementary number of match points to their opponents: i.e. the difference from $2(n – 1)$.

Each pair, whether playing NS or EW, can be said to receive two match points in respect of each pair playing the same way with a worse score, and one match point in respect of each pair with an equal score.

Measure of competition

Because scoring is by ranks, it is possible to quantify the amount of competition between two pairs. There are four cases.

(i) Pairs encounter one another

Consider pairs 1 to 8 in the above example. There are $2(n – 1)$ (=12) match points at stake, to be divided between pairs 1 and 8 according to the outcome of the encounter. Pair 8 can influence pair 1's match point score to the extent of ± 6.

Hence the amount of competition between pair 1 and pair 8 on the board is 6, or in general $(n – 1)$.

(ii) Pairs play in the same direction

Consider pairs 1 and 2 in the above example. Pair 1's match point score includes a contribution of 2 match points because they achieved a better score than pair 2. The influence of pair 2 on pair 1's score is ± 1 match point.

Hence the amount of competition between pair 1 and pair 2 on the board is 1.

(iii) Pairs play in opposite directions

Consider pairs 1 and 10 in the above example. By achieving a good score (+100) pair 10 have thereby enhanced the match point score of pair 1, who receive a contribution of two match points for scoring more than pair 10's opponents.

As in case *(ii)*, the performance of pair 10 can affect the score of pair 1 by ± 1 match point. The difference is that pairs 1 and 10 are not competing on the board, on the contrary their interests temporarily coincide.

Hence the amount of competition between pair 1 and pair 10 on the board is -1.

(iv) Board not played by both pairs
Evidently if a board is not played by a pair (e.g. because there is a half table) the amount of competition is zero.

Notation

Let

the number of pairs in a competition be	P
the number of boards in play be	N
the number of times each board is played be	n
the number of tables in play be	T
the number of boards in each set be	k
the number of rounds be	r

$P = 2T$ or $P = 2T + 1$, according to whether the number of pairs P is even or odd.

The total number of hands played is $Trk = Nn$,

A movement is *complete* if all the pairs play all the boards, when
$P=2T,\ n = T,\ N = kr$.

All the tables in Part I represent *complete* movements.

Competition analysis

Consider two pairs, numbered i and j, with $i < j$.
Let

- a_{ij} be the number of boards where i and j encounter one another;
- b_{ij} be the number of boards which i and j play in the same direction;
- c_{ij} be the number of boards which i and j play in opposite directions, but not as a direct encounter;
- d_{ij} be the number of boards not played by i and j (d_{ij} is always zero in a complete movement);
- s_{ij} be the amount of competition between i and j.

$$N = a_{ij} + b_{ij} + c_{ij} + d_{ij}$$
$$s_{ij} = (n - 1)a_{ij} + b_{ij} - c_{ij}$$

Mitchell Movement (Table I and IIa)

Tables I and IIa are square as well as complete, and therefore
$r = n = T;\ N = kn.$
If i and j are both stationary pairs, or both moving pairs,
$a_{ij} = c_{ij} = d_{ij} = 0;\ b_{ij} = N,\ s_{ij} = N$
If i is stationary and j is moving,
$a_{ij} =k;\ b_{ij},= d_{ij} = 0;\ c_{ij} = N - k;$
$s_{ij} = (n - 1)k - (N - k) = nk - N = 0,$ since $n = nk$.
Hence there is no net comparison between a NS pair and an EW pair; the two sets of players are decoupled from each other; there are effectively two separate competitions proceeding simultaneously, and it is appropriate to issue two results lists and to reward an EW winner and a NS winner.

Skip Movement (Table IIb)

The situation is more complicated for the skip movement of Table IIb.
As before $s_{ij} = N$ if i and j are both moving or both stationary.
If i is stationary and j is moving, there are three cases, depending on whether i and j encounter one another 0, 1 or 2 times, and the value of s_{ij} is $-N$, 0 or $+N$. There is now competition between some NS pairs and some EW pairs, and the two sets of players are no longer decoupled from each other.

For example, referring to Table IIb, if pair 9 happen to be strong players and pair 13 happen to be weak players, pair 1 are at a disadvantage because they have to encounter pair 9 twice and pair 13 not at all. Pair 5 are at a corresponding advantage and might be able to ride to victory on the backs of pair 9.

Balanced movements

In all the rectangular movements described in Part I, it will be found that the amount of comparison between a NS pair and an EW pair is never zero, but is sometimes positive and sometimes negative. Hence separate results lists for the two sets of players would not be fair, just as they would not be fair for the skip Mitchell of Table IIb. Moreover, most bridge players would prefer to think of themselves as participating in one large competition, and would prefer to see a single results list covering the whole field. Hence movements are required wherein the amount of competition between two pairs is independent as far as possible of where they happen to sit at the start.

There are P pairs, and thus there are $\frac{1}{2}P(P-1)$ values of s_{ij}. We require these values of s_{ij} to be all equal, or failing that, their dispersion as measured by their standard deviation ought to be as small as possible. If the s_{ij} are all equal and the movement is also complete, the movement is said to be *perfect*.

By summing over all pairs of pairs, we arrive at the following relations, valid for incomplete as well as complete movements:

$$\sum_{i<j} a_{ij} = nN,$$

$$\sum_{i<j} b_{ij} = \sum_{i<j} c_{ij} = n(n-1)N,$$

$$\sum_{i<j} s_{ij} = (n-1)\sum_{i<j} a_{ij} + \sum_{i<j} b_{ij} - \sum_{i<j} c_{ij} = n(n-1)N,$$

$\overline{s_{ij}} = n(n-1)N / \frac{1}{2}P(P-1)$ where $\overline{s_{ij}}$ is the average amount of competition between two pairs.

For a *complete* movement in which $P = 2T$ and $n = T$, the expression for $\overline{s_{ij}}$ simplifies to

$$\overline{s_{ij}} = N(T-1) / (2T-1). \qquad (1)$$

For a *perfect* movement, all the values of s_{ij} are equal, so that $\overline{s_{ij}}$ must be integral, and therefore N must be a multiple of $(2T-1)$.
Perfect movements do exist, but are not discussed in this paper.

Switching

The movements of Part I can be modified so that a single list of results can be produced which is not grossly unfair. Certain boards at certain tables are switched so that the stationary pairs play EW and the moving pairs play NS. The process is often called 'arrow switching' because boards have a large arrow painted on them to indicate the north. For example, Table VI is derived from Table I by switching the last round.

Table VI

Round	Table Number 1			2			3			4			5			6			7		
1	1	*A*	8	2	*B*	9	3	*C*	10	4	*D*	11	5	*E*	12	6	*F*	13	7	*G*	14
2	1	*B*	14	2	*C*	8	3	*D*	9	4	*E*	10	5	*F*	11	6	*G*	12	7	*A*	13
3	1	*C*	13	2	*D*	14	3	*E*	8	4	*F*	9	5	*G*	10	6	*A*	11	7	*B*	12
4	1	*D*	12	2	*E*	13	3	*F*	14	4	*G*	8	5	*A*	9	6	*B*	10	7	*C*	11
5	1	*E*	11	2	*F*	12	3	*G*	13	4	*A*	14	5	*B*	8	6	*C*	9	7	*D*	10
6	1	*F*	10	2	*G*	11	3	*A*	12	4	*B*	13	5	*C*	14	6	*D*	8	7	*E*	9
7	9	*G*	1	10	*A*	2	11	*B*	3	12	*C*	4	13	*D*	5	14	*E*	6	8	*F*	7

The values of s_{ij} instead of being either 0 or 7*k*, as they are for Table I, are now 0 or 3*k* or 4*k*. The distribution table for the s_{ij} is:

Value of s_{ij}	Example pairs	Number of pairs of pairs
0	1, 9	7
3*k*	1, 2	42
4*k*	1, 8	42
Standard deviation = 1.05*k*		

This is reasonably well balanced and is regarded as satisfactory. Any further switching becomes counter productive; for example, if Table I is switched on round 6 as well as round 7, the distribution table for the s_{ij} becomes:

Value of s_{ij}	Example pairs	Number of pairs of pairs
-*k*	1, 3	28
3*k*	1, 2	14
4*k*	1, 9	28
8*k*	1, 12	21
Standard deviation = 3.3*k*		

This is clearly ill balanced, and inferior to Table VI. One way of improving Table VI still further is to switch one board of each set on each of the last *k* rounds. For example if *k* = 4,

switch the first board of each set in round number 4,
switch the second board of each set in round number 5,
switch the third board of each set in round number 6,
switch the fourth board of each set in round number 7.

The distribution table for the s_{ij} is then:

Value of s_{ij}	Number of pairs of pairs
12	70
16	21
Standard deviation = 1.70	

The movement is approaching perfection, but it is seldom used, because of the difficulty in making sure that all the players switch the right boards at the right time.

How much to switch?

A rough and ready rule is to switch about one eighth of the boards in a Mitchell-type movement. Suppose that q out of N boards are switched, the switch always being made simultaneously at all tables. We assume also, as has been indicated by the above examples, that q is small compared to N.

Then, considering two stationary pairs i and j, it is likely that

$$a_{ij} = 0;\ b_{ij} = N - 2q;\ c_{ij} = 2q;\ s_{ij} = N - 4q.$$

The $2q$ boards which i and j play in opposite directions comprise the q boards which i play switched and the q boards which j play switched. In most cases the two sets of q boards are distinct.

Since s_{ij} is a little less than ½N (cf. equation (1)), it follows that q should be a little greater than $N/8$, so that the majority of values of s_{ij} should be close to the mean. Since all values of s_{ij} are integral, and switching does not alter the parity of s_{ij}, a first order approximation is as much as can reasonably be expected.

In movements such as the extended Mitchell (Table IV) and larger movements not considered here, the situation is less easily analysed, and a computer program is needed to calculate the optimum amount of switching. In a bridge club where there is little at stake, the rough justice which is offered by switching on the last round, or the last two rounds if there are 12 or more rounds, is acceptable. In some international competitions, however, movements have been prepared by computer, which involve irregular switching – each competing pair being given individual itineraries. No accusations of unfairness can then be levelled at the organisers.

Publications

The standard manual on bridge movements is 'Duplicate Bridge Movements' by Frank Farrington and published by the English Bridge Union, although not on sale to the public*. Farrington's manual lists dozens of different types of movement, many of which raise interesting mathematical questions. *Bridge Magazine* and other bridge periodicals carry articles on movements from time to time. Bridge club secretaries usually purchase sets of cards which indicate the movements for different numbers of tables – many of the movement cards which are commercially offered are seriously unbalanced.

I have been unable to trace any serious mathematical study of the subject, and I hope that this introductory article may stimulate others to build a corpus of theory. The ideas of duality in Part I, and of quantifying and attempting to equalise the amount of competition in Part II, have not previously been aired in print, although they have been floating round the bridge fraternity for several years.

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* Farrington's manual has been out of print for many years and was replaced in 1992 by *The EBU Manual of Duplicate Bridge Movements* edited by John Manning (ISBN 0 9506279 1 7). John paid particular attention to the questions of balance and arrow switching. Movement cards produced by the EBU are balanced as far as possible. The Movement Manual and other movement cards are available from the EBU Bridge Shop – 01296 397851 or email bridge.shop@ebu.co.uk .